METROLINK

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A14.4

Groundborne Noise
Numerical Modelling
Method FINDWAVE

Contents

Environmental Impact Assessment Report Volume 5
Appendix 14.4 Tunnel Boring Machine Numerical Modelling using Findwave Methodology

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14. Tunnel Boring Machine Numerical Modelling using Findwave® Methodology

This Appendix to Chapter 14 of the Environmental Impact Assessment Report (EIAR) provides information concerning the assumptions which have been made in the modelling of ground-borne noise and vibration from the operation of the MetroLink project.

14.1 Overview of Findwave®

The model is the finite-difference time-domain (FDTD) model *FINDWAVE®*, proprietary to Rupert Taylor Ltd.

FINDWAVE® is a finite difference time-domain numerical model for computing the propagation of waves in visco-elastic media. While it can be used to solve any acoustical or other 3-dimensional wave propagation problem its principal use is for the modelling of railway noise and vibration.

FINDWAVE® is also capable of modelling the vibration of railways at grade or in underground tunnels, including the transmission of ground-borne noise from the tunnels to the ground surface and into buildings.

The railway implementation of *FINDWAVE®* contains two principal (mutually interacting) modules, the train module, and the track/structure/environment module. The train module represents the train as a stack of damped masses and springs representing the rail vehicle. The excitation is provided from an input file containing either a measured or assumed vertical rail head profile, together with the gravitational effect of the rolling train. The train moves in the model and the location of the contact patch for each wheel is constantly advancing (re-entering the model at the front when it goes beyond the end). The interaction between the contact patch and the rail is transferred to the relevant fixed rail elements using polynomial interpolation as explained in the report.

The track/structure/environment module models the dynamic behaviour of the track and structure supporting the train, and the medium surrounding it, e.g. soil or air, together with structures below or above ground level. The structures concerned are represented as cells in a 3-dimensional orthogonal grid, each cell being assigned density, Lamé constants and loss factor.

14.2 Input Data

For the train module, the input data required are:

- Train formation
- Vehicle length
- Bogie pivot spacing
- Bogie wheelbase
- Vehicle body mass
- Vehicle secondary suspension stiffness
- Secondary suspension damping
- Sprung mass of bogie per wheel
- Stiffness of primary suspension
- Primary suspension damping
- Unsprung mass per wheel
- Rail mass per metre length

Environmental Impact Assessment Report Volume 5
Appendix 14.4 Tunnel Boring Machine Numerical
Modelling using Findwave Methodology

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- EI of rail

- Hertzian contact stiffness

For each layer of rail support:

- Stiffness of rail support

- Rail support damping

- Rail support stiffness

- Train speed profile

Rail roughness is read from a file of rail height in millimetres, in steps of not less than 5mm.

14.3   Wheel/Rail Roughness

The combined wheel/rail roughness assumption used is plotted in Figure 1 and is a 1/3 octave band spectrum, with the r.m.s. amplitude in the band centred on a wavelength of 2m being 30dB re 1 micron, decreasing with wavelength at the rate of 15 dB per decade. This assumption is representative of the combined wheel/rail roughness spectrum found on mass transit systems. It was derived following studies of the wheel and rail roughness measurements in the Rupert Taylor library, which were made on the District and Circle Line at the old Westminster and Circle Line Station, in the Channel Tunnel, in the Liverpool Loop, on Stockaryd-Lidnas and Helsingborg lines in Sweden and on wheelsets at the Ealing Common Depot of London Underground and on class 307 wheels at Birkenhead North Depot. There are, however, systems in existence which achieve a roughness spectrum of smaller amplitude, usually through a programme of pre-emptive rail grinding, or by the existence of favourable track characteristics.
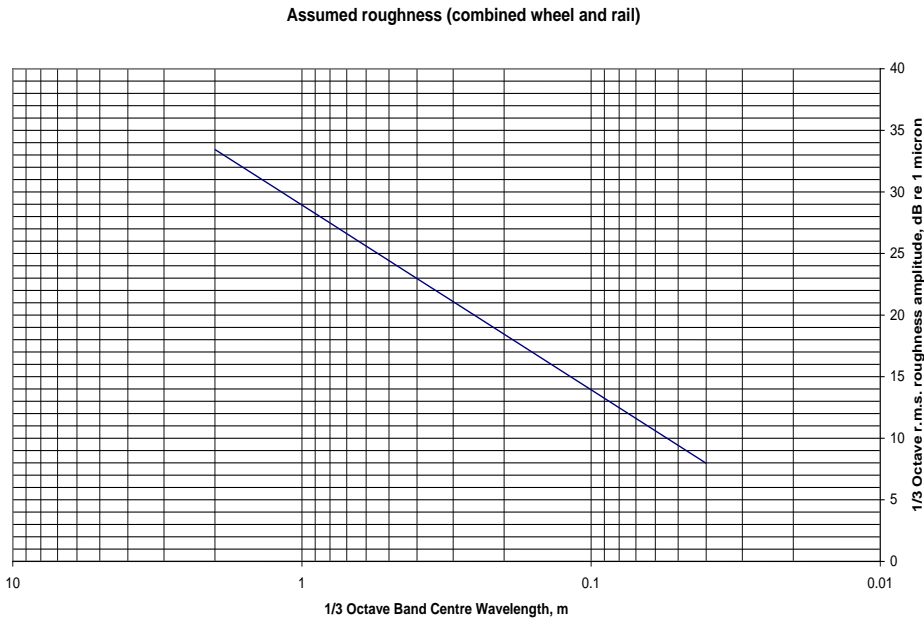
Assumed roughness (combined wheel and rail)

Diagram 1 Combined Wheel/Rail Roughness Spectrum

**Environmental Impact Assessment Report Volume 5
Appendix 14.4 Tunnel Boring Machine Numerical
Modelling using Findwave Methodology**

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14.4   Material Characteristics

The following assumptions concerning soil characteristics were used in the models.

The properties assigned to the materials modelled were as follows. The modulus assumptions are relevant to the extremely small strains involved in ground borne noise and vibration and are not necessarily the same as those used for civil engineering purposes. The property D is the compressive modulus, given by:

$$D = 2G(1-\sigma)/(1-2\sigma)$$

where σ is Poisson's ratio and G is shear Modulus.

Table 14.1 Properties of Materials in the Model

Material	Shear Modulus, $Gmax$, GPa	Compression Modulus, D, GPa	Density, ρ kg/m³	Loss factor η
Soil layers	0.45	3.26	2,350	0.05
Limestone	2.26	1.37	2,700	0.05

14.5   Output

Output data is stored as ASCII data files, suitable for importation into standard spreadsheets such as Excel. The primary output is the velocity or displacement of any software-selectable element in the model for each time step, which may be transformed into the frequency domain and into 1/3 octave or octave spectra. The time domain files may be stored as .WAV files, capable of replay as audible sound through the sound card of a computer.

Selected cross-sections of the model, showing the instantaneous displacement of the elements of the system, can be viewed at any time. For demonstration purposes, a sequence of cross sections can be stored during the running of the program for subsequent replay, after completion of the run, as a sequence showing progress of wave propagation through the cross section.

To perform an acceptably accurate prediction of levels of ground-borne noise in buildings above underground railways it is necessary to take account of:

1. The dynamic properties, dimensions, and speed of the rolling stock, and the roughness profile of the wheels;

2. The dynamic properties of the track and its support system, and the roughness profile of the rail running surface;

3. The characteristics of the tunnel;

4. The characteristics of the surrounding soil or rock, and of the ground above; and

5. The characteristics of the building.

All these parameters determine the behaviour of the complete system, which amounts to a complex array of elastic materials, through which wave propagation takes place.

Environmental Impact Assessment Report Volume 5
Appendix 14.4 Tunnel Boring Machine Numerical Modelling using Findwave Methodology

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By creating a representation of the system of elastic materials as a three-dimensional array of discrete cells, each possessing the four fundamental properties of an elastic medium[1], namely shear modulus, compression modulus, density and loss factor, and applying the rules of wave propagation in elastic media, it is possible to represent the real-life behaviour of the system, subject only to the accuracy of the parameters used and the limitations of the size of the cells in the array. For an array with cell sizes tending to zero, and assuming exact knowledge of the dynamic parameters, this approach will precisely represent the real-life behaviour of the system.

Application of the rules of wave propagation in elastic media is done by means of the wave equation. Leaving aside damping temporarily, and assuming initially a homogeneous isotropic medium, the wave equation states that the force acting on a small piece of elastic material will be due to two effects. Firstly, change of volume with distance will produce a force in that direction proportional to the compressive modulus of the material and the rate of change of volume with distance (i.e. the force is proportional to the pressure gradient). Secondly, change of shear angle with distance will produce a force in the direction of the shear (at right angle to the distance direction) proportional to the shear modulus of the material and the rate of change with distance.

14.6   The Algorithms

The wave equation in differential form is as follows:

$$(1) \qquad \mu\left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + \frac{\partial^2 \xi}{\partial z^2}\right) + (\lambda + \mu)\left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \eta}{\partial x \partial y} + \frac{\partial^2 \zeta}{\partial x \partial z}\right) = \rho \frac{\partial^2 \xi}{\partial t^2}$$

for the x axis, with corresponding equations for the y and z axes, where x, y, z and ⬜⬜⬜⬜⬜⬜⬜are displacements in three orthogonal axes;⬜ λ and μ are Lamé constants and ρ is the density. The Lamé constant μ is also known as the shear modulus, G. The Lamé constant λ is also known as the coefficient of dilatation and is given by:

$$\lambda = \frac{2\sigma G}{(1 - 2\sigma)}$$

where σ is Poisson's ratio.

Equation (1) can be stated in finite difference form by replacing the differential operator with the approximation:

$$\frac{\partial \xi}{\partial x} \approx (x[i][j][k] - x[i\text{-}1][j][k])/\Delta x (2)$$

For $\Delta x \rightarrow 0$ these two forms are identical.

For a homogeneous, isotropic medium with a finite value for Δx, Δy and Δz, elastic wave propagation can be computed using the finite difference substitution of equation (2)

Effectively, the process is as follows, for each axis, i, j and k. The example given is for axis i. Each point p(i,j,k) lies at the corner of a rectangular cell and is assigned a mass equal to one eighth of the sum of the eight contiguous

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[1] these can be different for each direction

Environmental Impact Assessment Report Volume 5
Appendix 14.4 Tunnel Boring Machine Numerical Modelling using Findwave Methodology

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cells as well as a displacement and velocity. The displacement and velocity is interpolated for each intermediate "virtual" point p(i+d,i+d,k+d) where d=0 or 0.5.

1. Compute pressure gradient
2. Compute shear force gradient
3. Accelerate p(i,j,k) by $\Delta v = F/\rho \, \Delta t$ where F is the sum of the force 1 & 2 and ρ is the density assigned to the point and v is the point velocity.
4. Displace p(i,j,k) by $\Delta x = \Delta v * \Delta t$ where x is the point displacement and t is one time step.
5. repeat from step 1

The geometric part of wave propagation is completely represented by this process. Further terms are required to represent damping. Of several possible terms, the inclusion of a coefficient by which the velocity is multiplied produces a loss factor which decreases within increasing frequency (and gives rise to an excess attenuation per unit distance which is independent of frequency). A viscous damping term can be used, by including a force proportional to acceleration multiplied by a coefficient. However, many materials exhibit hysteretic damping, or damping with other types of frequency dependence. To model these effects, it is necessary to include an algorithm which implements Boltzmann's strain history method where:

$$s(t) = D_1\varepsilon(t) - \int_0^\infty \varepsilon(t - \Delta t)\varphi(\Delta t)d(\Delta t)$$

where $\varphi(\Delta t) = \dfrac{D_2}{\tau} e^{-\Delta t / \tau}$ is an after-effect function, D_2 is a constant and τ is a relaxation time. D_1 is a modulus, $s(t)$ is stress and $\varepsilon(t)$ is strain. By combining several after-effect functions with different values of D_2 and τ any relationship between loss factor and frequency may be represented. Note that in the frequency domain the integral has a real and imaginary part, with the result that the value of the modulus is reduced by the inclusion of the relaxation terms. Depending on the choice of the constants and relaxation times, the stiffness of a resilient element will be frequency-dependent, and the value of D_1 must be adjusted at the same time that D_2 and τ are selected to give the required dynamic stiffness. This method has been implemented in the version of *FINDWAVE®* used for this study.

In principle the entire system can be represented by appropriate coding of cells to represent the vehicle body, suspension, bogies, wheels etc. In the present implementation, however, the train is represented as a set of stacks of lumped masses and springs, since the unsprung mass of wheels is by far the most influential parameter of the vehicle, and coupling between wheels, between axles and between bogies is normally insignificant.

For each wheel, the stack is as follows:

1. Mass, ρ_3, equal to one-eighth of the vehicle body mass for a four-axle vehicle.

2. Spring of stiffness K_2 and damping rate (Ns/m) C_2 one quarter of the secondary suspension.

3. Mass, ρ_2, equal to one quarter of the sprung bogie mass

4. Spring of K_1 and damping rate (Ns/m) C_1 equal to that of the primary suspension per wheel.

5. Mass, ρ_1, equal to the half the unsprung mass per axle.

6. Spring, K_h, with stiffness equal to the Hertzian contact spring stiffness assumed to be 1.2 GN/m.

The finite difference algorithm applied to the stack of masses and springs is as follows:

Environmental Impact Assessment Report Volume 5
Appendix 14.4 Tunnel Boring Machine Numerical Modelling using Findwave Methodology

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1. Compute compression Δz_h on Hertzian contact spring from difference between the displacement of the wheel mass and the displacement of the rail, minus the displacement of the roughness profile.

2. Accelerate unsprung mass by $\Delta v_1 = -K_h \Delta z_h / \rho_1 \Delta t$ where K_h is the stiffness of the Hertzian contact spring and ρ_1 is the unsprung mass.

3. Accelerate the unsprung mass by $\Delta v_g = -g \Delta t$ where g is gravitational acceleration.

4. Accelerate rail cell by $\Delta v_r = K_h \Delta z_h / \rho_r \Delta t$ where K_h is the stiffness of the Hertzian contact spring and ρ_r is the mass of the rail cell

5. Compute the compression of the primary suspension as the difference between the displacement of the sprung mass and the unsprung mass.

6. Accelerate sprung mass by $\Delta v_2 = -\{K_1 \Delta x_1 + C_1(v_2 - v_1)\}/\rho_2 \Delta t$ where K_1 is the stiffness of the primary suspension spring.

7. Accelerate the sprung mass by $\Delta v_g = -g \Delta t$ where g is gravitational acceleration.

8. *Accelerate unsprung mass by $\Delta v_1 = \{K_1 \Delta x_1 + C_1(v_2 - v_1)\}/\rho_1 \Delta t$.*

9. Compute the compression Δz_2 of the primary suspension as the difference between the displacement of the vehicle body mass and the sprung mass.

10. Accelerate vehicle body mass by $\Delta v_2 = -\{K_2 \Delta x_2 + C_2(v_3 - v_2)\}/\rho_3 \Delta t$ where K_2 is the stiffness and C_2 the damping rate of the secondary suspension spring and ρ_3 is the vehicle body mass.

11. Accelerate the vehicle body mass by $\Delta v_g = -g \Delta t$ where g is gravitational acceleration.

12. Accelerate sprung mass by $\Delta v_2 = \{K_2 \Delta x_2 + C_2(v_3 - v_2)\}/\rho_2 \Delta t$.

13. Displace the rail cell by $\Delta z_r = v_r \Delta t$

14. Displace the unsprung mass by $\Delta z_1 = v_1 \Delta t$

15. Displace the sprung mass by $\Delta z_2 = v_2 \Delta t$

16. Displace the vehicle body mass by $\Delta z_3 = v_3 \Delta t$

17. Compute new velocities and displacements for all cells in the structure/environment module

18. Repeat from (1)

The rail is represented as a line of cells with properties which give it the mass per unit length and bending stiffness appropriate to the rail section concerned.

The location of the contact patch between wheel and rail moves at the speed of the train. The displacement of the rail, which is computed at steps equal to the longitudinal cell size, is determined for the precise position of the contact patch by polynomial interpolation. This interpolated displacement is used to compute the compression of

Environmental Impact Assessment Report Volume 5
Appendix 14.4 Tunnel Boring Machine Numerical Modelling using Findwave Methodology

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the contact spring. The force produced by the contact spring is applied not only to the wheel, but also to the two ends of the rail cell in proportion to the distance of the contact patch from the opposite end of the cell.

For cases where the dynamic properties vary significantly, for example a solid component surrounded by air, manipulation is necessary to achieve correct results at the boundary of the solid and the air. The *FINDWAVE®* implementation achieves this by staggering the grid so that cell masses, elastic and damping constants are defined for points $p(i+\frac{1}{2},j+\frac{1}{2},k+\frac{1}{2})$ while displacements and velocities are computed for points $p(i,j,k)$ and interpolated for points $p(i+\frac{1}{2},j+\frac{1}{2},k+\frac{1}{2})$. Volumes and shear angles are computed for eight sub-sections of each cell.

14.7   Boundaries

For modelling finite objects fully surrounded by space, the boundaries can be represented by assigning zero-valued elastic moduli to the space provided that the acoustic load of the air in an airspace can be neglected. If radiation into air is to be modelled, or if an infinite or semi-infinite medium such as the ground is required, it is necessary to minimise the effect of reflections from the boundaries. For a train tunnel, where distances to be modelled are small compared with the length of the train, the z-axis boundaries are dealt with by creating a model exactly one rail vehicle (or unit of several coupled rail vehicles) in length, and then connecting the ends of the model together to create an infinitely long train. This is done by copying the cell displacements and velocities from one end of the model to the other end at the end of each time-step.

For the other boundaries in the x- and y-axes, the potential problem of spurious reflections from model boundaries is overcome by the use of an impedance matching technique. This effectively assigns to the cells which are required to be non-reflective on the boundaries of the model the properties of a massless viscous damper such that:

$$\frac{\eta K''}{\omega} = -\left(\rho c + \frac{D(\xi_o - \xi_{-1})}{\rho \Delta x v_0} \Delta t \right)$$

where η is the loss factor (dimensionless), K'' is the imaginary part of a complex spring stiffness in which the real part is zero, ω the angular frequency, ρc the characteristic impedance of the medium, ξ_0 and ξ_{-1} are the displacements of cell points 0 and -1 where the boundary is at cell 0, ρ is the density of the cell contents and v_0 is the velocity of cell 0. Over 95% absorption is achieved across the spectrum.

14.8   Model Input Data

The only input data required for the model are the masses of each cell, plus the shear modulus and the compression modulus, and the loss factor. A rail/wheel roughness profile is also required. Otherwise, all secondary parameters such as wave speeds, impedances etc. are automatically generated by the finite difference algorithm. The only other input relates to methods of approximating actual structure shapes using the orthogonal grid.

The output of the model consists of a file containing the displacement and/or velocity of one or more selected cells.

The time steps used are of the order of 30 to 60 microseconds, and the model is run for 65536 or 131072 steps to give a signal length of 1 second.

The resulting discrete time series can then be subjected to discrete fourier transformation to yield frequency spectra.

Note that, whereas in the acoustical analogy, the impedance of air varies little (except close to sources such as points), so that in most cases power is proportional to velocity squared, in elastic media, velocity transfer functions do not directly convey information about power transmission, and velocity at the receiver, in a low impedance

Environmental Impact Assessment Report Volume 5
Appendix 14.4 Tunnel Boring Machine Numerical Modelling using Findwave Methodology

JACOBS
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medium, can be higher than velocity near the source, in a high impedance medium, even when there are power losses between the source and the receiver.

14.9   Validation

The finite difference algorithm is validated by creating models of structures for which algebraic solutions are available and comparing the eigenfrequencies and decay rates. For Timoshenko beams, plates, thin and thick cylinders the eigenfrequencies are correctly predicted.